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Non-standard intersections of S-branes in $D = 11$ supergravity

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Abstract: We construct new intersecting S-brane solutions in 11-dimensional supergravity which do not have supersymmetric analogs. They are obtained by letting brane charges to be proportional to each other. Solutions fall into two categories with respect to whether there is a non-diagonal term to be cancelled in the field equations or not. In each case we show that they can be constructed by using a simple set of rules which is similar to the harmonic function rule of the usual static p-branes. Furthermore, we study an intersection where the Chern-Simons term makes a non-zero contribution to the field equations. We show that this configuration has a singularity like other S-branes.

Keywords: M-Theory, p-branes, D-branes.

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Contents

1. Introduction

S-branes are a particular class of D-branes [1] where along the time-like direction Dirichlet boundary condition is applied [2]. Therefore they exist only for a moment in time. Recently effort to understand these objects increased considerably (see, e.g. [3]–[18]) since they are interesting both as time-dependent backgrounds and for tachyon condensation. They may also be useful in establishing the dS/CFT duality [19].

Although the name *S-brane* first appeared in [2] and more examples were obtained subsequently in [3, 4], such supergravity solutions were studied before [20]–[24] where the primary motivation was cosmology. Additionally, let us note that similar solutions were also investigated by relating them to Toda-like systems [25, 10]. (See [26] for a review and more references.)

After the construction of single S-brane solutions [2, 3, 4] a natural next step was to consider their intersections. In [4], multiply charged SDp/SD$(p-2)$ and SDp/SD$(p-4)$ brane solutions were constructed. In [6] all possible orthogonally intersecting S-brane solutions of $D = 11$ supergravity theory corresponding to supersymmetric M-brane intersections were obtained. Later in [14] the S-brane intersection rules were generalized to arbitrary dimensions. In [6, 14] it was observed that all solutions can be obtained by a simple procedure of multiplying the brane and the transverse directions by certain powers of two hyperbolic functions of time. This is the S-brane analog of the harmonic function rule of the usual p-branes [27]–[30].

The S-brane intersections that are obtained in [6] are in 1-1 correspondence with the supersymmetric intersections of M-branes. (For instance, there is $SM2 \perp SM5(1)$ instead of $M2 \perp M5(1)$.) In this paper we will call those solutions *standard* and show that actually there exist more intersections without supersymmetric duals. Since, S-branes anyway break all supersymmetry, there is a priori no reason for neglecting these cases.

In standard intersections in $D = 11$ each brane has 3 free parameters and the intersection rules derived in [6] are found by assuming that all parameters are independent from each other. However, we will show in this paper that relaxing this condition allows other intersections as well.[1] In particular, if brane charges are proportional to each other, then more solutions are possible.

The organization of this paper is as follows. In the next section we will begin construction of non-standard intersections in which the Chern-Simons term doesn't effect the field equations. They fall into two categories with respect to whether there is a non-diagonal term to be cancelled in the field equations or not. In the second one we only have a condition on the magnitudes of the charges, whereas in the first we also need oppositely charged branes in the solution. In each subclass we find that solutions can be constructed by using a simple set of rules which is similar to the set of rules found for standard intersections [6] upto some small modifications. In section 3 we will analyze an example, $SM2 \perp SM5 \perp SM2(-1)$, where the Chern-Simons term makes a non-trivial contribution to the field equations and imposes some constraints unlike any other solution studied in the literature so far. The equations are quite hard to solve analytically, however using numerical methods we will show that there is still singularity. We will conclude in section 4 with some comments and possible future directions.

2. Intersections with $F \wedge F = 0$

The bosonic action of the 11-dimensional supergravity can be written as

$$S = \int d^{11}x \left(\sqrt{-g}R - \frac{1}{2.4!}\sqrt{-g}F^2 - \frac{1}{6}F \wedge F \wedge A \right) \tag{2.1}$$

where the last term is the Chern-Simons term. The equations of motion are given by

$$R_{AB} = \frac{1}{2.3!}F_{ACDE}F_B{}^{CDE} - \frac{1}{6.4!}g_{AB}F^2\,, \tag{2.2}$$

$$d * F = \frac{1}{2}F \wedge F\,. \tag{2.3}$$

We consider a metric of the following form

$$ds^2 = -e^{2A}dt^2 + \sum_k e^{2C_k}\, ds_k^2 + e^{2D}\, d\Sigma_{n,\sigma}^2 \tag{2.4}$$

where ds_k^2 (for $k = 1, 2\ldots$) is the metric on the d_k dimensional flat space. $d\Sigma_{n,\sigma}^2$ is the metric of the n-dimensional unit sphere $\sigma = 1$, unit hyperbola $\sigma = -1$ or flat space $\sigma = 0$. The metric functions depend only on t.

In this section we are interested in solutions where $F \wedge F = 0$ and thus the 4-form field equations reduce to $dF = d * F = 0$. The spatial transverse space to any SM2-brane

[1]This possibility was mentioned in [14] and the solutions that we will construct in section 2.1 correspond to block-orthogonal intersections in the Toda approach [10].

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is 7-dimensional and if we denote the closed volume-form of this space with Vol_7, then the 4-form field of the i'th SM2-brane can be written as

$$F_i = q_i * \mathrm{Vol}_7\,, \tag{2.5}$$

where $*$ is the Hodge dual with respect to the full metric. Similarly, for each SM5-brane, the space-like transverse space is 4-dimensional. Defining Vol_4 to be the volume-form of this space, the 4-form field corresponding to the j'th SM5-brane is equal to

$$F_j = q_j\,\mathrm{Vol}_4\,. \tag{2.6}$$

It is easy to see that we have $dF = 0$ and $d * F = 0$ for both cases.

One can simplify the Ricci tensor by fixing t-reparametrization invariance so that

$$A = \sum_k d_k C_k + nD. \tag{2.7}$$

With respect to the orthonormal frame $E^0 = e^A dt$, $E^{\alpha_k} = e^{C_k} dy^{\alpha_k}$ and $E^\theta = e^D e^\theta$, where e^θ is an orthonormal frame on $\Sigma_{n,\sigma}$ and with the above gauge choice the Ricci tensor becomes [6]

$$\begin{aligned}
R_{00} &= e^{-2A}\left[-A'' + A'^2 - \sum_k d_k C_k'^2 - nD'^2\right] \\
R_{\alpha_k\beta_k} &= e^{-2A}\left[C_k''\right]\delta_{\alpha_k\beta_k} \qquad k = 1,2\ldots \\
R_{\theta_1\theta_2} &= e^{-2A}\left[D''\right]\delta_{\theta_1\theta_2} + \sigma(n-1)e^{-2D}\delta_{\theta_1\theta_2},
\end{aligned}$$

where all derivatives are with respect to the time coordinate t.

The curvature scalar is

$$R = e^{-2A}\left[2A'' - A'^2 + \sum_k d_k C_k'^2 + n\,D'^2\right] + \sigma n(n-1)\,e^{-2D}\,. \tag{2.8}$$

Now we are ready to ask which configurations are allowed? There are 3 restrictions coming from our choice of the metric (2.4) and the field equations (2.2) and (2.3): (i) The dimension of the transverse space without time should be at least two, i.e., $n \geq 2$. This is necessary in order to be able to solve the (00) component of the Ricci tensor. This rules out $SM5 \perp SM5(0)$, $SM5 \perp SM5(1)$, $SM2 \perp SM5(2)$ and $SM2 \perp SM5(-1)$. (ii) In this section we assume that $F \wedge F = 0$. This eliminates single $SM2 \perp SM5(2)$ pair. (iii) Our Ricci tensor is diagonal and therefore there shouldn't be any term at the right hand side of the Einstein's equation (2.2) coming from the $F_{AMNP}F_B{}^{MNP}$ contraction. This excludes single $SM5 \perp SM5(4)$ and single $SM2 \perp SM2(1)$ pairs.

Taking into account these restrictions we are left with:

$$\text{Standard Pairs}: SM2 \perp SM2(0), SM2 \perp SM5(1), SM5 \perp SM5(3) \tag{2.9}$$

$$\text{Non-Standard Pairs}: SM2 \perp SM2(-1), SM2 \perp SM5(0)\,. \tag{2.10}$$

By *standard* we mean intersections which have supersymmetric analogs. All possible cases when each brane is making a standard intersection with others, have been constructed in [6]. In the next subsection we will construct all allowed combinations between the two groups listed above. The constraints (ii) and (iii) can be surmounted by adding more pairs which contributes in the same non-diagonal direction but with opposite sign. We leave the investigation of these to the subsection 2.2.

2.1 Intersections without any non-diagonal terms

In this part we will first construct $SM2 \perp SM2(-1)$ and $SM2 \perp SM5(0)$ solutions and then give rules to construct intersections containing more branes. Formulas will be given in the most general form whenever possible. For $SM2 \perp SM2(-1)$ we have $n = 4$ and (-1) means that they don't have any common tangent direction. (They share only the moment of their existence.) Let the first SM2-brane be located at (x_1, x_2, x_3) with charge q_1 and another SM2-brane be located at (x_4, x_5, x_6) with charge q_2. Then, the metric (2.4) becomes,

$$ds^2 = -e^{2A}dt^2 + e^{2C_1}\left(dx_1^2 + dx_2^2 + dx_3^2\right) + e^{2C_2}\left(dx_4^2 + dx_5^2 + dx_6^2\right) + e^{2D}d\Sigma^2_{4,\sigma}\,. \tag{2.11}$$

We have already explained above how to solve the 4-form field equation (2.3). From the spatial components of Einstein's equations (2.8) we get:

$$C_1'' = -\frac{q_1^2}{3}e^{-6C_2-8D+2A} + \frac{q_2^2}{6}e^{-6C_1-8D+2A} \tag{2.12}$$

$$C_2'' = \frac{q_1^2}{6}e^{-6C_2-8D+2A} - \frac{q_2^2}{3}e^{-6C_1-8D+2A} \tag{2.13}$$

$$D'' + 3\sigma e^{-2D+2A} = \frac{q_1^2}{6}e^{-6C_2-8D+2A} + \frac{q_2^2}{6}e^{-6C_1-8D+2A}\,. \tag{2.14}$$

Let us define function g as:

$$(n-1)g = A - D\,. \tag{2.15}$$

Now, we see that this system of equations is solvable if we choose $C_1 = C_2 = -\frac{f}{3}$, $D = g + \frac{2}{3}f$ and $A = 4g + \frac{2}{3}f$ together with $q_1^2 = q_2^2 = 2Q^2$. Note that this is consistent with our gauge condition (2.7). Then,

$$f'' = Q^2 e^{-2f} \tag{2.16}$$

$$g'' = -\sigma(n-1)e^{2(n-1)g}\,. \tag{2.17}$$

These two equations have integrability conditions,

$$\left(f'\right)^2 + Q^2 e^{-2f} = M_1^2 \tag{2.18}$$

$$\left(g'\right)^2 + \sigma e^{2(n-1)g} = M_2^2\,. \tag{2.19}$$

The relation between integration constants (M_1, M_2) is found from the R_{00} component of the Ricci tensor to be:

$$M_1^2 \sum_k q_k^2 = 2n(n-1)M_2^2 Q^2\,. \tag{2.20}$$

If we define $H = e^{2f}$ and $G_{n,\sigma} = e^{-2(n-1)g}$, from equations (2.16) and (2.17) we get

$$H = \frac{Q^2}{M_1^2} \cosh^2 \left[M_1 \left(t - t_0 \right) \right] . \tag{2.21}$$

and

$$G_{n,\sigma} = \begin{cases} M_2^2 \sinh^2 [M_2(n-1)\, t] & \sigma = -1 \quad \text{(hyperbola)}, \\ M_2^2 \cosh^2 [M_2(n-1)\, t] & \sigma = 1 \quad \text{(sphere)}, \\ e^{2M_2(n-1)\, t} & \sigma = 0 \quad \text{(flat)}, \end{cases} \tag{2.22}$$

where the translational invariance in time is used to remove one constant in (2.22). The final form of the $SM2 \perp SM2(-1)$ solution is

$$\begin{aligned} ds^2 &= H_1^{-1/3} H_2^{1/6} \left(dx_1^2 + dx_2^2 + dx_3^2 \right) + H_1^{1/6} H_2^{-1/3} \left(dx_4^2 + dx_5^2 + dx_6^2 \right) + \\ &\quad + H_1^{1/6} H_2^{1/6} G_{4,\sigma}^{-4/3} \left(-dt^2 + G_{4,\sigma}\, d\Sigma_{4,\sigma}^2 \right) \qquad (2.23) \\ F &= q_1 * (dx_4\, dx_5\, dx_6 \Omega_4) + q_2 * (dx_1\, dx_2\, dx_3 \Omega_4) \,. \qquad (2.24) \end{aligned}$$

Here $d\Sigma_{4,\sigma}^2$ is the metric on the unit sphere, the unit hyperbola or flat space and Ω_4 is its volume-form. The hodge dual is with respect to the full metric (2.23). Let us remind that for this intersection $n = 4$ and we have the conditions $H_1 = H_2 = H^2$ and $q_1^2 = q_2^2 = 2Q^2$, which implies $M_1^2 = 6M_2^2$. The purpose of writing the solution as above with H_1 and H_2 is that the same form appears in all other intersections. The construction of $SM2 \perp SM5(0)$ follows the same steps. Let q_1 and q_2 be the charges of the SM2 and SM5-branes respectively. Then the solution can be written as

$$\begin{aligned} ds^2 &= H_1^{-1/3} H_2^{-1/6} \left[dx_1^2 \right] + H_1^{-1/3} H_2^{1/3} \left[dx_2^2 + dx_3^2 \right] + H_1^{1/6} H_2^{-1/6} \left[dx_4^2 + \cdots + dx_8^2 \right] + \\ &\quad + H_1^{1/6} H_2^{1/3} G_{2,\sigma}^{-2} \left[-dt^2 + G_{2,\sigma}\, d\Sigma_{2,\sigma}^2 \right] , \qquad (2.25) \\ F &= q_1 * (dx_4 \ldots dx_8\, \Omega_2) + q_2 (dx_2\, dx_3\, \Omega_2) \,. \qquad (2.26) \end{aligned}$$

The SM2-brane is oriented along (x_1, x_2, x_3), and SM5-brane is oriented along $(x_1, x_4, \ldots, x_8)$ hyperplanes. Again we have $H_1 = H_2 = H^2$ and $q_1^2 = q_2^2 = 2Q^2$, where H and G are defined in (2.21) and (2.22) with $n = 2$. This solution was also given in [10].

It is straightforward to generalize this method to obtain other intersections. By assuming that each term at the right-hand side of the Ricci tensor (2.8) has the same exponential, one gets ratios between the charges. Then, choosing all C_k's to be a suitable multiple of a function f (some of them might be zero) and using (2.15), all spatial components of Einstein equations are reduced to the equations (2.16) and (2.17). From the time component of the Ricci tensor the integrability condition (2.20) is obtained.

The maximum number of SM5 and SM2-branes one can use is 4 and 8 respectively. For their combinations the maximum numbers are (1 SM5+ 6 SM2), (2 SM5 + 4 SM2) and (3 SM5+ 3 SM2). There are at least 18 non-standard intersections which involve only SM2's, 2 intersections with only SM5's and 30 intersections with both. We constructed all intersections which have upto 5 branes and couple of typical ones in the 5'th order. After doing this we found that all of them follow the same pattern and can be constructed by using a simple set of rules (we expect these to be valid in the remaining ones too) which are:

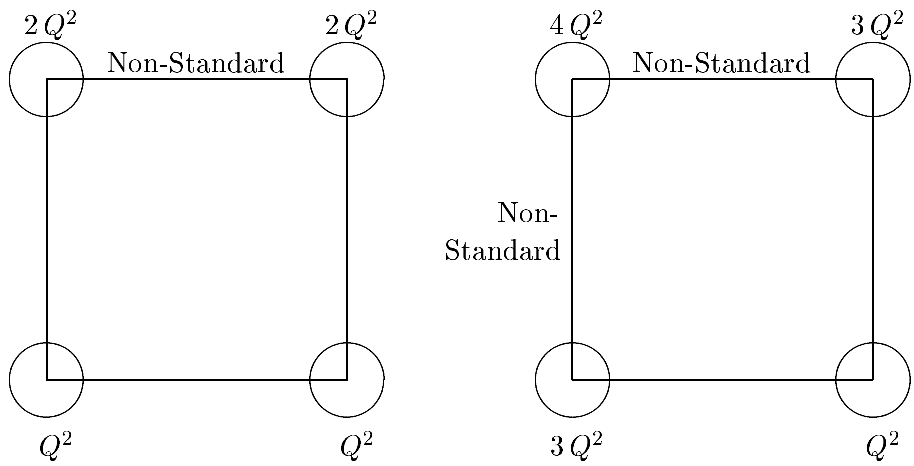

Figure 1: The ratios of charge squares depend on whether there is a brane making more than one non-standard intersection in the configuration (which is at most 1) or not (rule (iii)).

(i) Each additional brane to the system should make either standard (2.9) or non-standard (2.10) intersection with every other brane.

(ii) Always n, the dimension of the transverse space without time, has to satisfy $n \geq 2$.

(iii) For charges, we have two distinct cases. If a system contains an SM2-brane which makes more than one non-standard intersection,[2] then the charge of that brane is $4Q^2$. In this case, the charges of branes making only one non-standard intersection is $3Q^2$. Otherwise, each brane making one non-standard intersection carries charge $2Q^2$ (figure 1). In both cases a brane making only standard intersections has charge Q^2. For example, in the solution where SM2-branes are located at (x_1, x_2, x_3), (x_4, x_5, x_6) and (x_4, x_7, x_8) with charges $(q_1,\ q_2,\ q_3)$ respectively, we have $q_1^2 = 4Q^2$, $q_2^2 = q_3^2 = 3Q^2$. Whereas, in another triple intersection of SM2's with locations (x_1, x_2, x_3), (x_4, x_6, x_7) and (x_1, x_4, x_5) charges are $q_1^2 = q_2^2 = 2Q^2$ and $q_3^2 = Q^2$.

(iv) For each brane its H_k function is given as

$$H_k = H^{q_k^2/Q^2} \tag{2.27}$$

where H is defined in (2.21). In the first example given in rule (iii) we have $H_1 = H^4$ and $H_2 = H_3 = H^3$. On the other hand, in the second example $H_1 = H_2 = H^2$ and $H_3 = H$.

(v) The metric is obtained by multiplying the brane and the transverse directions for k'th brane by appropriate powers of H_k which are

$$\text{For i'th SM2-brane:} \begin{cases} \text{Brane direction} & H_i^{-1/3} \\ \text{Transverse direction} & H_i^{1/6} \end{cases} \tag{2.28}$$

[2]In solutions of this subsection there can be at most one brane making two non-standard intersections and it has to be an SM2-brane, e.g. (x_1, x_2, x_3) with (x_4, x_5, x_6), (x_1, x_7, x_8). One can add (1 SM5+2 SM2) or 4 SM2's to this system.

$$\text{For j'th SM5-brane:} \begin{cases} \text{Brane direction} & H_j^{-1/6} \\ \text{Transverse direction} & H_j^{1/3} \end{cases} \tag{2.29}$$

(vi) Upto the H-functions, the overall transverse space takes the form

$$G_{n,\sigma}^{-\frac{n}{(n-1)}} \left[-dt^2 + G_{n,\sigma}\, d\Sigma_{n,\sigma}^2\right] \tag{2.30}$$

where $d\Sigma_{n,\sigma}^2$ is the metric on the unit sphere, the unit hyperbola or flat space.

(vii) The 4-form field strength of the configuration is

$$F = \sum_i F_i + \sum_j F_j \tag{2.31}$$

where F_i's are for SM2's given in (2.5) and F_j's are for SM5's given in (2.6). With these rules one can construct all possible non-standard intersections of SM-branes. Let us mention that it is possible to have intersections where all intersections are standard, yet the overall intersection is non-standard. This happens, for example in $SM2 \perp\ SM2 \perp SM2(-1)$ intersection where branes are located at (x_1, x_2, x_3), (x_2, x_4, x_6) and (x_1, x_4, x_5) with charges $q_1^2 = q_2^2 = q_3^2 = Q^2$. We would like to emphasize that there may be more that one configuration with a given number of branes, e.g. there are three configurations for $SM2 \perp\ SM2 \perp SM2(-1)$.

Let us now compare the intersections found here and those standard ones obtained in [6]. To begin with, their construction rules are very similar to each other. In both of them solutions can be constructed from two hyperbolic functions H and G defined in (2.21) and (2.22) using rules (v) and (vi). The main difference is the number of integration constants. In non-standard solutions there are only 3 parameters, namely M_1(or M_2), Q^2 and t_0. In fact, the constant M_1(or M_2) can be removed by a scaling $t \to t/M_1$ followed by further scalings of x_i coordinates or by redefinition of the charge Q, if necessary. Therefore, these solutions depend only on 2 constants. On the other hand, there are $(3x-1)$ free constants in standard intersections where x is the number of branes. In the standard intersections any brane can be removed from the system by certain scalings [6]. However, in our case since branes are indistinguishable from each other this can not be done. Another difference is the rule (iv). In the standard intersections all H_k's are proportional to the square of $\cosh(t - t_0)$, whereas here the power depends on charge ratios. In common with the standard ones, there is always a singularity since at least one of the metric functions vanishes as $t \to \pm\infty$. In some of them, like $SM2 \perp SM5(0)$, the singularity is milder in the sense that, the curvature scalar (2.8) is zero. As pointed out in [6] when the overall tranverse space has dimension greater than 2, i.e. $(n = 2 + m)$, then one can flatten (2.30) as

$$G_{n,\pm}^{-\frac{n}{(n-1)}} \left[-dt^2 + G_{n,\pm}\, d\Sigma_{n,\pm}^2\right] \to dz^2 + G_{n-1,\pm}^{-\frac{(n-1)}{(n-2)}} \left[-dt^2 + G_{n-1,\pm}\, d\Sigma_{n-1,\pm}^2\right]. \tag{2.32}$$

This procedure can be repeated m times. The coordinate z can be used to reduce the solution to type-IIA theory. Then, applying S and T-duality transformations, one can obtain these type of intersecting S-brane solutions in type-IIA and IIB supergravities.

2.2 Intersections with non-diagonal terms

In this subsection we will explore intersections containing $SM2 \perp SM2(1)$, $SM5 \perp SM5(4)$ and $SM5 \perp SM2(2)$. These were excluded in the previous subsection because either there is a non-diagonal term appearing at the right-hand side of the Einstein's equations or $F \wedge F$ is not zero. However, these difficulties can be overcome by adding extra branes so that these undesired terms disappear. To classify solutions of this type is much harder than those of the previous section, since there can be more than one non-diagonal direction or there can be more than two pairs of branes cancelling the same non-diagonal term. We will mainly focus on intersections containing $SM2 \perp SM2(1)$, and investigate the problem for upto 2 non-diagonal directions with only two pairs for every cancellation and comment on what happens in other cases. We explicitly checked the validity of those rules given in section 2.1 upto 3 additional branes (but we strongly believe they are valid for others too) and found that they are still respected; there is only a some small modification in rule (iii) which we will indicate. Hence, it is enough to specify Q^2 in (2.21) and the relations between the charges to describe a solution using rules (v), (vi) and (vii).

Now, let us consider $SM2 \perp SM2(1)$ with SM2's located at (x_1, x_2, x_3) and (x_1, x_2, x_4) with charges (q_1, q_2) respectively. Then, there is a non-diagonal term coming from the contraction $F_{3ABC}F_4{}^{ABC}$ in equation (2.2). Let us add two more SM2-branes at (x_3, x_5, x_6) and (x_4, x_5, x_6) with charges (q_3, q_4). Now, there is only 1 non-diagonal direction R_{34} for the Ricci tensor and if we choose $q_1q_2 = -q_3q_4$, then the non-diagonal terms cancel each other. Following the method of the previous section, we find that the spatial components of Einstein's equations (2.8) are reduced to two integrable equations (2.16) and (2.17) that we obtained before provided that $q_1^2 = q_4^2$ and $q_2^2 = q_3^2$. Results given in (2.20), (2.21) and (2.22) are still valid with $Q^2 = (q_1^2 + q_2^2)/2$ and $n = 4$. The metric and the 4-form can be obtained using rules (iv)–(vii) and it is:

$$
\begin{aligned}
ds^2 &= H^{-1/3}(dx_1^2 + \cdots + dx_6^2) + H^{2/3}G_{4,\sigma}^{-4/3}\left(-dt^2 + G_{4,\sigma}\, d\Sigma_{4,\sigma}^2\right) \\
F &= q_1 * (dx_4 dx_5 dx_6 \Omega_4) + q_2 * (dx_3 dx_5 dx_6 \Omega_4) + \\
&\quad + q_3 * (dx_1 dx_2 dx_4 \Omega_4) + q_4 * (dx_1 dx_2 dx_3 \Omega_4)\,.
\end{aligned}
\tag{2.33}
$$

The form of the metric coincides with that of $SM2 \perp SM2(-1)$ given in (2.23). Another way of looking at this system is to consider the first and the second branes as a single brane with charge $(q_1^2 + q_2^2)$. (Similarly third and fourth with charge $q_3^2 + q_4^2 = q_1^2 + q_2^2$.) Since in this quartet each brane makes one non-standard intersection (we count the non-diagonal intersections between q_1 and q_2 (or q_3 and q_4) as standard when applying (rule (iii)) , this explains why the charge of a brane making only standard intersections should be $Q^2 = (q_1^2 + q_2^2)/2$ by rule (iii). If this view point is correct than adding two SM2's at (x_1, x_5, x_7) and (x_1, x_6, x_8) should be different than adding SM2's at (x_1, x_5, x_7) and (x_2, x_6, x_8). In each case additional branes make standard intersections with the brane quartet but among themselves they differ. Indeed, in the first one additional branes have charges $q_5^2 = q_6^2 = (q_1^2 + q_2^2)/2$ whereas in the second they carry $q_5^2 = q_6^2 = q_1^2 + q_2^2$ as expected.

After looking at many examples we realized that, in addition to the restrictions arising from rules (i) and (ii), branes can be added to the system (2.33) only when they satisfy the following exclusion principle: *After the addition of branes, the pair* (q_1, q_2) *should receive the same number of non-standard intersections as that of the pair* (q_3, q_4). For example, we can't add just a single SM2 along (x_1, x_7, x_8) to (2.33), because it makes 2 non-standard intersections with the second pair but none with the first. This rule can be understood as follows: To cancel the non-diagonal term we need $q_1 q_2 = -q_3 q_4$. However, from the rule (iii) we know that making extra non-standard intersections effects charges, and therefore to keep the relation valid, we need the above principle.

Now let us state the modification in rule (iii). *When there are additional branes, in applying rule (iii) to the brane quartet (2.33), only external non-standard intersections should be counted.* To illustrate, let us add an SM2 at (x_3, x_7, x_8). Although it respects the symmetry between the pairs (q_1, q_2) and (q_3, q_4) and therefore this addition is possible, it makes non-standard intersections only with (q_2, q_4). This system is solvable when $(q_1^2, q_2^2, q_3^2, q_4^2, q_5^2) = (q_1^2, 3q_1^2, q_1^2, 3q_1^2, 4q_1^2)$ which is compatible with rule (iii) if we don't count internal non-standard intersections of these 4 branes. We can add a sixth brane to this system at (x_2, x_5, x_7) which makes standard intersections only and therefore its charge should be $q_6^2 = q_1^2$. One can add at most five SM2-branes to the system (2.33) and there are 15 different possibilities. For SM5's the maximum number is three and there are 3 possibilities. And finally, for combinations we have 7 cases and maximum numbers are (2 SM5+ 2 SM2) and (1 SM5 + 3 SM2).

Intersections with two non-diagonal Ricci components work similarly. Let SM2-branes be located at $(x_1, x_2, x_3), (x_1, x_2, x_4), (x_2, x_3, x_5), (x_2, x_4, x_5)$ with charges (q_1, q_2, q_3, q_4). Now, to cancel R_{34} and R_{15} we need $q_1 q_2 = -q_3 q_4$ and $q_1 q_3 = -q_2 q_4$. These imply $q_1^2 = q_4^2$ and $q_2^2 = q_3^2$ which also solve Einstein's equations. In this solution $Q^2 = q_1^2 + q_2^2$ and it is given as,

$$
\begin{aligned}
ds^2 &= H^{-1/3}\left(dx_1^2 + dx_3^2 + dx_4^2 + dx_5^2\right) + H^{-2/3} dx_2^2 + H^{1/3} G_{5,\sigma}^{-5/4}\left(-dt^2 + G_{5,\sigma}\, d\Sigma_{5,\sigma}^2\right) \\
F &= q_1 * (dx_4 dx_5 \Omega_5) + q_2 * (dx_3 dx_5 \Omega_5) + q_3 * (dx_1 dx_4 \Omega_5) + q_4 * (dx_1 dx_3 \Omega_5)\,. \qquad (2.34)
\end{aligned}
$$

This is a tighter system if we want to add more branes, since in addition to the above exclusion principle we should also consider (q_1, q_3) and (q_2, q_4) as pairs. We see that additions are possible only when both $(q_1,\ q_4)$ and $(q_2,\ q_3)$ brane pairs receive the same amount of non-standard intersections. For example, we can't put a single brane at (x_1, x_6, x_7) since it makes a non-standard intersection with q_4 but a standard one with q_1. If we add an SM2 at (x_6, x_7, x_8) which makes non-standard intersections with all others, then it should carry charge $q_5^2 = 4(q_1^2 + q_2^2)/3 = 4Q^2/3$. A sixth brane can be added at (x_1, x_5, x_6) which makes only standard intersections with charge $q_6^2 = (q_1^2 + q_2^2)/3$. (If the fifth brane was absent then this brane would have charge $(q_1^2 + q_2^2)$ because members of the system (2.34) makes only standard intersections between themselves.) To the system (2.34) one can add at most 3 SM2's with 7 distinct ways, and 3 SM5's with 3 possibilities. There are 7 cases for their combination with maximum numbers (1 SM5+3 SM2) and (2 SM5 + 1 SM2).

Nothing new happens in intersections containing $SM2 \perp SM5(2)$ or $SM5 \perp SM5(4)$. Lets consider the configuration with branes at (x_1, x_2, x_3), $(x_1, \ldots, x_6)$, (x_1, x_7, x_8),

$(x_1, x_4 \ldots, x_8)$, with charges $(q_1, \ldots, q_4)$. To get rid of the $F \wedge F$ term we need $q_1 q_2 = -q_3 q_4$ which is in accordance with Ricci tensor conditions $q_1^2 = q_4^2$ and $q_2^2 = q_3^2$. We have $Q^2 = (q_1^2 + q_2^2)/2$. Similarly, for $SM5 \perp SM5(4)$ pairs located at $(x_1, \ldots, x_6)$, $(x_1, \ldots, x_5, x_7)$, $(x_1, \ldots, x_4, x_6, x_8)$, $(x_1, \ldots, x_4, x_7, x_8)$ with charges $(q_1, \ldots, q_4)$ we find conditions $q_1 q_2 = -q_3 q_4$ and $q_1 q_3 = -q_2 q_4$ together with $q_1^2 = q_4^2$ and $q_2^2 = q_3^2$ to solve the field equations. In this solution $Q^2 = q_1^2 + q_2^2$.

It is straightforward to construct intersections with more number of non-diagonal Ricci tensors and we expect them to obey the same principles. However, since there are more algebraic conditions to satisfy it becomes quite complicated to analyze and after a point direct construction becomes easier than trying to decide how to apply rules. For example, let us add two SM2's located at six branes located at $(x_2, x_3, x_5), (x_1, x_4, x_6)$ with charges (q_5, q_6) to the configuration (2.33). There are 3 non-diagonal terms along R_{34}, R_{26} and R_{15}. For cancellation we need $q_1 q_2 = -q_3 q_4$, $q_3 q_5 = -q_2 q_6$ and $q_1 q_5 = -q_4 q_6$, which are consistent with $q_1^2 = q_4^2$, $q_2^2 = q_3^2$ and $q_5^2 = q_6^2$ that are required for Einstein's equations. The solution is given by $Q^2 = (q_1^2 + q_2^2 + q_5^2)/2$. In this case (q_1, q_2, q_5) triplet behaves like a single brane. As a second case let us consider two additional branes to the system (2.33) at $(x_1, x_2, x_7), (x_5, x_6, x_7)$ with charges (q_5, q_6). This time our non-diagonal directions are R_{34}, R_{37} and R_{47}. Now, cancellations and field equations are both satisfied only when $Q^2 = q_1^2 = \cdots = q_6^2$.

Another possibility to get rid of unwanted terms is to add more than one pair of branes. For example, consider 6 SM2-branes located at (x_1, x_2, x_3), (x_1, x_2, x_4), (x_3, x_5, x_6), (x_4, x_5, x_6), (x_3, x_7, x_8) and (x_4, x_7, x_8) with charges $(q_1, \ldots, q_6)$ respectively. Then, to cancel the non-diagonal term R_{34} we need $q_1 q_2 + q_3 q_4 + q_5 q_6 = 0$. The Ricci components are solved as before with conditions $q_1^2 + q_2^2 = q_3^2 + q_4^2 = q_5^2 + q_6^2$ and $q_1^2 + q_3^2 + q_5^2 = q_2^2 + q_4^2 + q_6^2$. These conditions can be solved simultaneously. One consistent choice is $q_6 = q_1 = (2 + \sqrt{3}) q_2$, $q_2 = q_5$, $q_4 = -q_3$ and $q_3^2 = 2(2 + \sqrt{3}) q_2^2$. In this intersection $Q^2 = (q_1^2 + \ldots + q_6^2)/12$. It seems that one can continue adding more branes. However, to satisfy all conditions gets harder.

Despite the difficulty in classifying all the solutions of this section, constructing any one of them is straightforward with our method. The physical properties of them are like those obtained in the previous section. Again none of the branes is separable and there are singularities. Once more, they depend only on two parameters (Q^2, t_0), but there is a little more freedom for charges since usually the proportionality of all charges is not required. Instead, some anti-branes are necessary in the system.

3. An intersection with non-vanishing $F \wedge F$ term

In this section we would like to investigate an intersection where the Chern-Simons term has a non-zero contribution to the 4-form field equation (2.3). In all the S-brane solutions constructed until now [2, 3, 4, 6] this term didn't play any role. For this purpose, let us consider two SM2-branes located at (x_1, x_2, x_3) and (x_4, x_5, x_6) and an SM5-brane located at $(x_1, \ldots, x_6)$. Our metric and 4-form field strength are

$$ds^2 = -e^{2A} dt^2 + e^{2C_1} \left(dx_1^2 + dx_2^2 + dx_3^2 \right) + e^{2C_2} \left(dx_4^2 + dx_5^2 + dx_6^2 \right) + e^{2D} \, d\Sigma_{4,\sigma}^2 \quad (3.1)$$

$$F = a(t)\overline{dt} \wedge \overline{dx_1} \wedge \overline{dx_2} \wedge \overline{dx_3} + b(t)\overline{dt} \wedge \overline{dx_4} \wedge \overline{dx_5} \wedge \overline{dx_6} + q\, d\Omega_4 \tag{3.2}$$

where coordinates with bar are in the orthonormal frame. The Bianchi identity $dF = 0$ is satisfied trivially. Again we use the gauge condition $A = 3C_1 + 3C_2 + 4D$ to simplify the Ricci tensor. This allows us to choose $A = D + 3g = -C_1 - C_2 + 4g$. We also define

$$p = ae^{-4C_1 - C_2}\,, \qquad r = be^{-4C_2 - C_1} \tag{3.3}$$

Note that $F \wedge F$ is not zero and from the 4-form field equation (2.3) we get two constraints:

$$p' = q\, r\, e^{6C_2} \tag{3.4}$$

$$r' = -q\, p\, e^{6C_1}\,. \tag{3.5}$$

Spatial components of the Ricci tensor gives

$$C_1'' = -\frac{1}{3}p^2\, e^{6C_1} + \frac{1}{6}r^2\, e^{6C_2} - \frac{q^2}{6}\, e^{6C_1 + 6C_2} \tag{3.6}$$

$$C_2'' = \frac{1}{6}p^2\, e^{6C_1} - \frac{1}{3}r^2\, e^{6C_2} - \frac{q^2}{6}\, e^{6C_1 + 6C_2} \tag{3.7}$$

$$g'' = -3\,\sigma\, e^{6g}\,. \tag{3.8}$$

Notice that when q is zero this system of equations reduces to equations (2.12)–(2.14) which we wrote in the previous section for $SM2 \perp SM2(-1)$ with $p = q_1$ and $r = q_2$. The last equation is decoupled from others and its solution is given in (2.22). However, the remaining two are non-linear equations and we couldn't solve it analytically. But, numerical techniques can be used to see the behaviour of the metric functions. The time component of the Ricci tensor is necessary for a consistent choice of the initial values and that gives,

$$-12(C_1')^2 - 12(C_2')^2 - 12C_1'C_2' + 24M_2^2 = p^2e^{6C_1} + r^2e^{6C_2} + q^2e^{6C_1 + 6C_2} \tag{3.9}$$

where M_2^2 comes from the integrability of the function g given in (2.19). In picking up initial values it is also useful to notice $2q(C_1'' - C_2'') = (pr)'$. Actually there are several options for initial values which change the asymptotic values of the functions p and r. However, we observed that the asymptotic behaviour of the metric functions e^{2C_1} and e^{2C_2} do not depend much on these choices and they always approach to zero as $t \to \infty$ which signals a singularity (figure 2).

4. Conclusions

In this paper, we have constructed a large number of orthogonally intersecting S-brane solutions in $D = 11$ supergravity which do not have supersymmetric analogs. They depend only on 2-parameters and there is no parameter to distinguish one brane from the other. It is remarkable that all of them can be obtained just by using two functions defined in (2.21) and (2.22) and a simple set of axioms which are the main results of

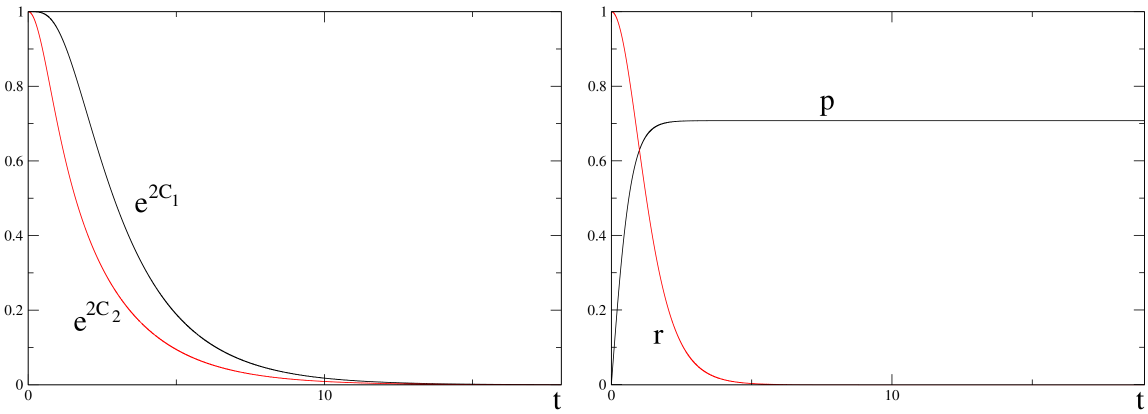

Figure 2: The metric and form functions with initial values $p(0) = C_1(0) = C_2(0) = C_1'(0) = C_2'(0) = 0$, $r(0) = 1$ and $q = 1$, $M_2^2 = 1/12$.

this paper. It is nice to see that despite the absence of supersymmetry both the solutions obtained in this paper and those standard intersections obtained before in [6] have similar construction rules which is a beautiful reflection of the structure of the $D = 11$ supergravity field equations. We also analyzed a solution where the Chern-Simons term has a non-trivial effect to the field equations. Unfortunately, as in other supergravity S-brane solutions there is a naked singularity. Recently there has been some progress in resolution of these singularities [9, 13, 17] by using gravitational backreaction of tachyon matter.

It seems that there is a close relationship between S-brane solutions and non-extremal p-brane solutions. The technique to obtain S-branes was used to construct black p-branes in [3] and the similarity of the solutions are obvious. It is known that upto Wick rotations one can sometimes map a cosmological solution into a p-brane solution (see for example [31, 22]). It would be nice to understand this connection for S-branes. Then, one can see what the S-brane analogs of monopole and wave solutions of the $D = 11$ supergravity are and study their intersections. Also it would be interesting to find out the static version of the $SM2 \perp SM2(-1)$ configuration. We expect that most of the solutions we obtained have static counterparts. For instance, non-supersymmetric but extremal $M2 \perp M5(0)$ solution is discussed in [32]. In addition, several non-extreme static brane configurations where charges and harmonic functions are not independent are found in [33, 34]. In these the common tangent directions are not Poincaré invariant.

Certainly more work is required in order to understand how to distinguish standard and non-standard intersections from each other. M-brane intersections can be divided into 3 classes by looking at their binding energies (see [35] for a review). A generalization of ADM mass for S-brane backgrounds is necessary to do such comparison. We hope that our solutions will be useful in this respect.

There are various possible generalizations of the solutions we found. First of all, instead of adding anti-pairs to obtain intersections that are worked out in section 2.2, one may seek a different ansatz for the 4-form field strength. Secondly, all our solutions are orthogonal and one may consider intersections with angles. Thirdly, one may assume curved world-volume for S-branes and study their intersections. Of course, when both the world-volume and the transverse space are curved equations may not be solvable in general [22]. And finally, our solutions depend only on time and one may try to find localized ones where solutions depend on other transverse directions as well.

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